Analyzing Piecewise Functions

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Objective:
Students will analyze attributes of a piecewise function including the area of a region bounded by a piecewise function and the *x*-axis.

Connections to Previous Learning:
Students should be familiar with the terms domain, range, slope, increasing/decreasing intervals, absolute minimum/maximum, finding function values, translation of functions, equations of lines, circles, absolute value, interval notation, and piecewise functions.

Connections to AP*:
AP Calculus Topic: Analysis of Functions

Materials:
Student Activity pages; no calculators

Teacher Notes:
The intent of this lesson is to connect the students' previously learned knowledge of functions with piecewise functions. This type of global lesson encourages retention of important topics such as transformations and area by revisiting them each time the student sees a new type of function. Since piecewise functions provide an important teaching tool in the Advanced Placement Calculus program, encourage the retention and understanding of these particular functions by exposing students to them early and often.

All area calculations can be determined using formulas for typical geometric shapes.

When the students are asked to determine the maximum or the minimum value of the function, explain that the question asks for the largest or the smallest *y*-value of the function and not for the coordinates of a point. The *x*-coordinate tells you where or when the maximum/minimum value occurs, while the *y*-coordinate tells you the maximum or minimum value of the function. The same maximum or minimum *y*-value may occur at more than one point.

Students should NOT use calculators in this lesson.

Teachers may want to make two transparency copies of the graphs so the reflections can be seen easily.

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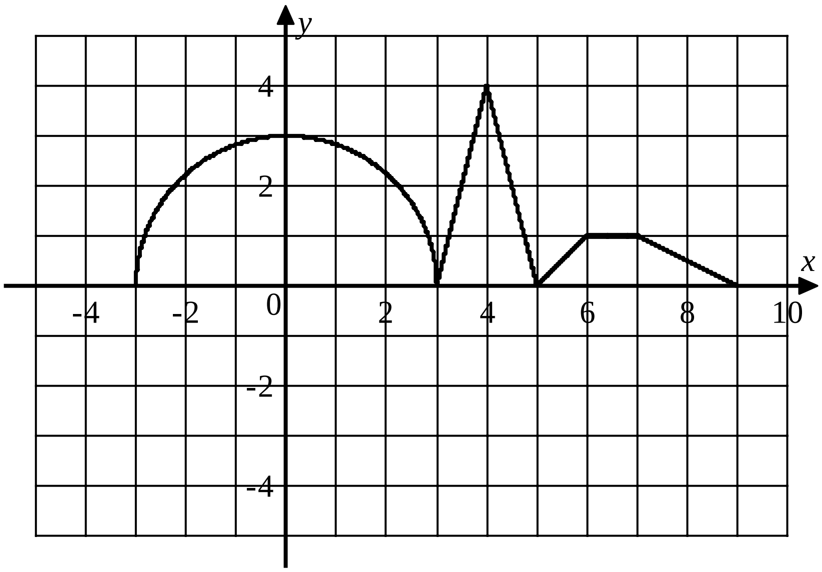
Use the graph of $y = f(x)$ and the table of values to answer the questions.



x	$f(x)$
-3	0
0	3
3	0
4	4
5	0
6	1
7	1
9	0

1. Classify each of the geometric figures formed between the graph of $f(x)$ and the x-axis.

2. What is the total area enclosed by the graph of $f(x)$ and the x-axis?

3. Give the equation for each line of symmetry, if one exists, on the following intervals of the graph of $f(x)$.

   [–3, 3] __________ [3, 5] ___________ [5, 9] ___________ [–3, 9] __________

4. For each of the intervals above, how do the lines of symmetry divide the area between the graph of $f(x)$ and the x-axis?

5. A vertical line $x = k$ divides the area enclosed between the graph of $f(x)$ and the x-axis into two equal parts. Which geometric figure does this line intersect?

6. If $g(x) = –f(x)$, determine the total area between the graph of $g(x)$ and the x-axis.

7. Classify each of the geometric figures formed between the graphs of $f(x)$ and $g(x)$.

8. What is the area enclosed by the graphs of $f(x)$ and $g(x)$?

9. If $h(x) = 2\,f(x)$, what is the total area between the graph of $h(x)$ and the x-axis?

   (Hint: The figure between $x = –3$ and $x = 3$ is now half of an ellipse with a semi-major axis of $a = 6$ and a semi-minor axis $b = 3$. The area of an ellipse is calculated by the formula $A = \pi ab$, where a and b are the semi-major and semi-minor axes lengths.)

10. If $r(x) = 4\,f(x)$, what is the total area between the graph of $r(x)$ and the x-axis?

11. Compare the area between the graph of $f(x)$ and the x-axis to the area between the graph of $p(x)$ and the x-axis, where $p(x) = \text{a}\,f(x)$, $a > 0$.

12. What is the total area bounded by the graph of $f(x)$, and the lines $y = –1$, $x = –3$, and $x = 9$?

13. If $q(x) = f(x) + 1$, what is the total area bounded by $q(x)$, the x-axis, $x = –3$, and $x = 9$?

14. On what intervals is $f(x)$ decreasing?

15. On what intervals is $f(x)$ increasing?

16. What is the absolute maximum value of f(x)?

17. What is the x-coordinate(s) where the absolute minimum value of $f(x)$ occurs?

18. If $g(x) = 2\,f(x)$, what is the absolute maximum value?

19. Write the equation for $f(x)$ using five equations.

    (Hint: Write an equation involving absolute value for the portion of the function between $x = 3$ and $x = 5$.)

$$f(x) = \begin{cases} \underline{\hspace{6cm}}, & \underline{\hspace{3cm}} \\ \underline{\hspace{6cm}}, & \underline{\hspace{3cm}} \\ \underline{\hspace{6cm}}, & \underline{\hspace{3cm}} \\ \underline{\hspace{6cm}}, & \underline{\hspace{3cm}} \\ \underline{\hspace{6cm}}, & \underline{\hspace{3cm}} \end{cases}$$

20. What is x when $f(x) = \frac{1}{2}$?

21. Determine the slope at each of the following points on the graph of $f(x)$. If the point is on the semicircle, the slope will be the same as the slope of the tangent line to the semicircle at that point.

(3.5, _____); slope = _________

(4.5, _____); slope = _________

(6.213, _____); slope = ________

$\left(7\frac{4}{5},\ \_\_\_\_\_\right)$; slope = ________

(–3, ______); slope = ________

(0, ______); slope = _________

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Answers:

1. semicircle; isosceles triangle; trapezoid
2. $\frac{9\pi}{2}+6\frac{1}{2}$
3. on [–3, 3], $x = 0$; on [3, 5], $x = 4$; on [5, 9] and on [–3, 9], no line of symmetry
4. divides the area in half
5. semicircle
6. $\frac{9\pi}{2}+6\frac{1}{2}$
7. circle, rhombus, irregular hexagon
8. $9\pi+13$
9. $9\pi+13$
10. $18\pi+26$
11. The area between $p(x)$ and the x-axis is the area between the graph of $f(x)$ and the x-axis multiplied by a.
12. $\frac{9\pi}{2}+18\frac{1}{2}$
13. $\frac{9\pi}{2}+18\frac{1}{2}$
14. $x\in[0, 3]$ [4, 5[[7, 9]
15. $x\in[-3, 0]$ [3, 4] [5, 6]
16. 4
17. –3; 3; 5; 9
18. 8

19. $f(x)=\begin{cases}\sqrt{9-x^2}, & -3\le x\le 3\\ -4\,|\,x-4\,|+4, & -3<x\le 5\\ x-5, & 5<x\le 6\\ 1, & 6<x\le 7\\ \dfrac{-1}{2}(x-9), & 7<x\le 9\end{cases}$

20. $x=\dfrac{\pm\sqrt{35}}{2}$; $4\dfrac{7}{8}$; $3\dfrac{1}{8}$; $5\dfrac{1}{2}$; 8

21. (3.5, 2); slope = 4 | (4.5, 2); slope = -4

(6.213, 1); slope = 0 | $\left(7\dfrac{4}{5},\dfrac{3}{5}\right)$; slope = $\dfrac{-1}{2}$

(–3, 0); slope does not exist | (0, 3); slope = 0